

Solutions

Name: _____

Work in groups to answer as many problems as you can. Ask questions if you get stuck.

1. Simplify the expressions, using exponent rules, leaving your answer in rational exponent form.

(a) $2x^2\sqrt{4x^6}$

$$\begin{aligned}
 &= 2x^2(4x^6)^{1/2} \\
 &= 2x^2 \cdot 4^{1/2} (x^6)^{1/2} \\
 &= 2x^2 \cdot 2x^3
 \end{aligned}$$

$4x^5$

Answer: _____

(b) $2m^2 \cdot 4m^{3/2} \cdot 4m^{-2}$

$$\begin{aligned}
 &= 2 \cdot 4 \cdot 4 \cdot m^2 \cdot m^{3/2} \cdot m^{-2} \\
 &= 2 \cdot 16 \cdot m^{3/2} m^{2-2} \\
 &= 32 \cdot m^{3/2} \cdot m^0
 \end{aligned}$$

$32m^{3/2}$

Answer: _____

(c) $(a^{1/2})^{3/2}$

$$= a^{1/2 \cdot 3/2}$$

$a^{3/4}$

Answer: _____

(d) $\frac{2x^{-7/4}}{4x^{4/3}}$

$$= \frac{2}{4} \cdot \frac{x^{-7/4}}{x^{4/3}} = \frac{1}{2} \cdot x^{-7/4 - 4/3} = \frac{1}{2} x^{-\frac{21}{12} - \frac{16}{12}}$$

$\frac{1}{2} x^{-37/12}$

Answer: _____

(e) $\sqrt{n^4}^3$

$$= ((n^4)^{1/2})^3 = (n^2)^3$$

n^6

Answer: _____

$$(f) \sqrt[5]{27p^6} = ((27p^6)^{1/3})^5 = (27^{1/3}(p^6)^{1/3})^5 = (3 \cdot p^2)^5 = 3^5(p^2)^5$$

Answer: $3^5 p^{10}$

$$(g) \frac{1}{\sqrt{25b^3}} = \frac{1}{((25b^3)^{1/2})^3} = \frac{1}{(25^{1/2}(b^3)^{1/2})^3} = \frac{1}{(5b^{3/2})^3} = \frac{1}{5^3(b^{3/2})^3} = 5^{-3} \cdot \frac{1}{b^9}$$

Answer: $5^{-3} \cdot b^{-9}$

$$(h) \sqrt[3]{a^8} = ((a^8)^{1/3})^2 = (a^{8/3})^2$$

Answer: $a^{16/3}$

$$(i) \frac{4x^2}{2x^{1/2}} = \frac{4}{2} \cdot \frac{x^2}{x^{1/2}} = 2 \cdot x^{2-1/2} = 2x^{3/2}$$

Answer: $2x^{3/2}$

$$(j) \frac{3x^{-1/3} \cdot 3x^{1/2} \cdot 7^{-1/3}}{3y^{-7/4}} = \frac{3 \cdot 3 \cdot x^{-1/2} \cdot x^{1/2} \cdot 7^{-1/3}}{3 \cdot y^{-7/4}} = \frac{3 \cdot x^{\frac{1}{2}-\frac{1}{2}} \cdot y^{7/4}}{7^{1/3}} = \frac{3 \cdot x^0 \cdot y^{7/4}}{7^{1/3}}$$

Answer: $\frac{3y^{7/4}}{7^{1/3}}$

2. Expand the following expressions using the distributive property.

$$\begin{aligned} \text{(a)} \quad & 3(2p - 5) + 2(3p - 3) \\ & = 3(2p) + 3(-5) + 2(3p) + 2(-3) \\ & = 6p - 15 + 6p - 6 \end{aligned}$$

Answer: 12p - 21

$$\begin{aligned} \text{(b)} \quad & x(x^2 - 2y) + 3x^2(x + 2y) \\ & = x(x^2) + x(-2y) + 3x^2(x) + 3x^2(2y) \\ & = x^3 - 2xy + 3x^3 + 6x^2y \end{aligned}$$

Answer: 4x³ + 6x²y - 2xy

$$\begin{aligned} \text{(c)} \quad & a(a + 2b - 3c) + 3c(a - 2b + 3c) - 2b(a - b - 3c) \\ & = a(a) + a(2b) + a(-3c) + 3c(a) + 3c(-2b) + 3c(3c) \\ & \quad + 2b(a) - 2b(-b) - 2b(-3c) \\ & = a^2 + 2ab - 3ac + 3ac - 6bc + 9c^2 - 2ab + 2b^2 + 6bc \end{aligned}$$

Answer: a² + 9c² + 2b²

$$\begin{aligned} \text{(d)} \quad & (b - c + d)(a - c - d) \\ & = b(a - c - d) - c(a - c - d) + d(a - c - d) \\ & = a(b) + b(-c) + b(-d) - c(a) - c(-c) - c(-d) \\ & \quad + d(a) + d(-c) + d(-d) \\ & = ab - bc - bd - ac + c^2 + cd + ad - cd - d^2 \end{aligned}$$

Answer: ab - bc - bd - ac + c² + ad - d²

$$\begin{aligned} \text{(e)} \quad & 3s(2t - 3r + 4u) - 2t(3s - r + 6u) \\ & = 3s(2t) + 3s(-3r) + 3s(4u) - 2t(3s) - 2t(-r) - 2t(6u) \\ & = 6st - 9rs + 12su - 6st + 2rt - 12tu \end{aligned}$$

Answer: -9rs + 12su + 2rt - 12tu

(f) $(x + y + z)(5z + 2y)$

$$= x(5z + 2y) + y(5z + 2y) + z(5z + 2y)$$

$$= x(5z) + x(2y) + y(5z) + y(2y) + z(5z) + z(2y)$$

$$= 5xz + 2xy + 5yz + 2y^2 + 5z^2 + 2yz$$

Answer: $5xz + 2xy + 7yz + 2y^2 + 5z^2$

(g) $p(q - 2r) - 2(rp + 7q)$

$$= p(q) + p(-2r) - 2(rp) - 2(7q)$$

$$= pq - 2pr - 2pr - 14q$$

Answer: $pq - 4pr - 14q$

(h) $2x^2(4xy - 5) - 8yx^3 + 9x^2$

$$= 2x^2(4xy) + 2x^2(-5) - 8yx^3 + 9x^2$$

$$= 8x^3y - 10x^2 - 8x^3y + 9x^2$$

Answer: $-x^2$

(i) $w + z(5w + 6 - z) + (z + 4w)(w + 6)$

$$= w + z(5w) + z(6) + z(-z) + z(w + 6) + 4w(w + 6)$$

$$= w + 5wz + 6z - z^2 + wz + 6z + 4w^2 + 24w$$

Answer: $25w + 6wz + 12z - z^2 + 4w^2$

(j) $5s + 6ts(r + 5t) + 5r(2s - 3t)(2s + 3t)$

$$= 5s + 6ts(r) + 6ts(5t) + 5r(2s - 3t)(2s) + 5r(2s - 3t)3t$$

$$= 5s + 6rst + 30st^2 + 10rs(2s - 3t) + 15rt(2s - 3t)$$

$$= 5s + 6rst + 30st^2 + 10rs(2s) + 10rs(-3t) + 15rt(2s) + 15rt(-3t)$$

Answer: $5s + 6rst + 30st^2 + 20rs^2 - 30rst + 30rst - 45rt^2$

$$5s + 6rst + 30st^2 + 20rs^2 - 45rt^2$$

Remember

• $A^2 - B^2 = (A - B)(A + B)$

• $A^2 + 2AB + B^2 = (A + B)^2$

• $A^2 - 2AB + B^2 = (A - B)^2$

3. Factorise the following expressions.

(a) $x^2 - y^2$

Answer: $(x - y)(x + y)$

(b) $w^8 - 2^4$

$$= (w^4)^2 - (2^2)^2$$

Answer: $(w^4 - 2^2)(w^4 + 2^2)$

(c) $w^2 + 2zw + z^2$

Answer: $(w + z)^2$

(d) $s^2 + t^2 - 2st$

$$= s^2 - 2st + t^2$$

Answer: $(s - t)^2$

(e) $2mn - 6m^2n + 4mn^3$

$$= 2(mn - 3m^2n + 2mn^3)$$

$$= 2m(n - 3mn + 2mn^2)$$

$$= 2mn(1 - 3m + 2mn)$$

Answer: $2mn(1 - 3m + 2mn)$

(f) $5p^2q - 10pq + 15p$

$$= 5(p^2q - 2pq + 3p)$$

$$= 5p(pq - 2q + 3)$$

$$5p(pq - 2q + 3)$$

Answer: _____

(g) $9jk^2l^3 + 6j^2l - 3jkl$

$$= 3(3jk^2l^3 + 2j^2l - jkl)$$

$$= 3j(3k^2l^3 + 2jl - kl)$$

$$= 3jl(3k^2l^2 + 2j - k)$$

$$3jl(3k^2l^2 + 2j - k)$$

Answer: _____

(h) $xy + 17x^2yz + 3xy^5z^4$

$$= x(y + 17xyz + 3y^5z^4)$$

$$= xy(1 + 17xz + 3y^4z^4)$$

$$xy(1 + 17xz + 3y^4z^4)$$

Answer: _____

(i) $a^2b^2 - c^2$

$$= (ab)^2 - c^2$$

$$(ab - c)(ab + c)$$

Answer: _____

(j) $x^4y^2z^6 - a^8b^6c^2$

$$= (x^2yz^3)^2 - (a^4b^3c)^2$$

$$(x^2yz^3 - a^4b^3c)(x^2yz^3 + a^4b^3c)$$

Answer: _____